

# Forest Hills Primary School

## Calculation Policy



The purpose of our Calculation Policy is to ensure consistency in the teaching of Mathematics throughout the school and to ensure that pupils develop efficient written and mental methods of calculation, underpinned by conceptual understanding.

## Calculation Policy

This policy provides an overview of the strategies used in our school to teach Mathematics, specifically the four operations, as defined within the National Curriculum in England: Mathematics Programme of Study.




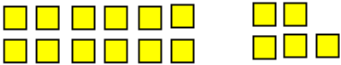
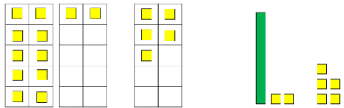


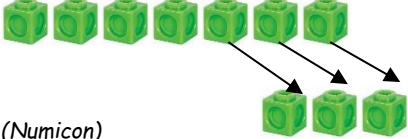
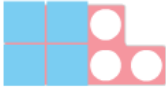

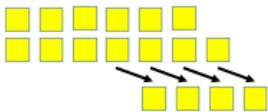
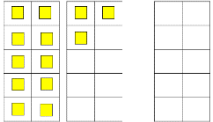





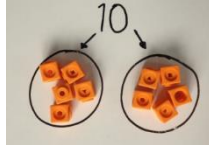


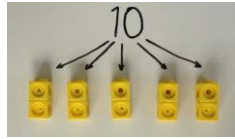
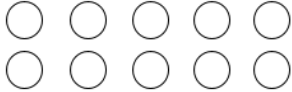
The progression of the four operations (+, -,  $\times$  and  $\div$ ) are shown across each of the primary year groups 1 - 6. This is a guide since children progress at different rates. Teachers should model strategies appropriate to the ability of the children they teach, regardless of their year group, whilst striving to achieve age related expectations at the end of the academic year.

At Forest Hills Primary School, we believe that children should be introduced to the processes of calculation through the **concrete, pictorial** and **abstract** (CPA) approach. Our children are introduced to calculation through practical activities, using **concrete** resources. As children develop their understanding of the underlying concepts and mathematical models, they develop ways of recording to support their thinking. In the first instance, this recording takes the form of **pictorial** representations. Over time, children learn how to use models and images to support their mental and informal written methods of calculation.

As children become more proficient in their use of mental methods, their informal written methods also become more efficient. Some recording takes the form of jottings, which are used to support children's thinking. More **abstract**, formal written methods are taught only when the child is able to use a wide range of mental calculation strategies and these are always underpinned by **concrete** and **pictorial** experiences.

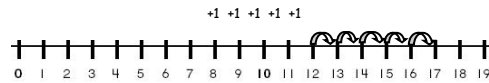
Our ultimate aim is for children to be able to select an efficient method to solve problems. Therefore, children will be encouraged to look at a calculation or problem and to determine the most appropriate method to choose - pictures, mental calculation with or without jottings or a formal, written method.

The end of year expectations in the National Curriculum shows the progression in children's use of calculation within the following strands 'Addition and Subtraction' and 'Multiplication and Division'. These end of year expectations will be achieved through the use of the following written methods of calculation.

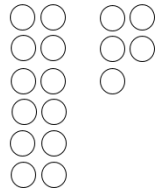
Year	Addition +	Subtraction -	Multiplication x	Division ÷
1	<ul style="list-style-type: none"> <li>♦ Add <b>one-digit and two-digit numbers</b> to 20 including zero.</li> <li>♦ Read, write and interpret mathematical statements involving addition (+) and equal (=) signs.</li> </ul>	<ul style="list-style-type: none"> <li>♦ Subtract <b>one-digit and two-digit numbers</b> to 20 including zero.</li> <li>♦ Read, write and interpret mathematical statements involving subtraction (-) and equal (=) signs.</li> </ul>	<ul style="list-style-type: none"> <li>♦ Begin to understand <b>multiplication</b> through doubling numbers and quantities.</li> <li>♦ Use arrays and sets of 'equal groups' to look at other multiples, e.g. <math>\times 5</math>.</li> </ul>	<ul style="list-style-type: none"> <li>♦ Begin to understand <b>division</b> through grouping and sharing small quantities.</li> </ul>
	<p>Addition of single digits:  <math>5 + 3 = 8</math>            ...using <b>concrete</b> equipment:              (Numicon)  </p> <p>Addition of two digit numbers to 20 and a one digit number:  <math>12 + 5 = 17</math>            ...using <b>concrete</b> equipment:            (Numicon)              (Dienes)              (Dienes and ten frames)                (Bead Strings)  </p>	<p>Subtraction of single digits  <math>7 - 4 = 3</math>            ...using <b>concrete</b> equipment:              (Numicon)  </p> <p>Subtraction of a one-digit number from a two-digit number to 20.  <math>13 - 4 = 9</math>            ... using <b>concrete</b> equipment:            (Numicon)              (Dienes)              (Ten frames)  </p>	<p>Doubling - linking to <math>\times 2</math>            Double 4 is 8 or <math>4 + 4 = 8</math> or <math>4 \times 2 = 8</math>            ... using <b>concrete</b> equipment:              (Numicon)  </p> <p>... using <b>pictorial</b> representations:  </p> <p>Use an array or equal groups to solve multiplication problems for multiples other than 2  <math>5, 3</math> times or <math>5 \times 3 = 15</math>            ... using <b>concrete</b> equipment            (Numicon)              I then use my 10s checker  </p>	<p>Sharing equally            Share 10 into 2 equal groups            ... using <b>concrete</b> equipment:              (Numicon)            Count how many are in each set = 5</p> <p>Model putting the 2s on top of the ten Numicon tile. How many 2s have I used? 5              ... using <b>pictorial</b> representations:  </p> <p>... using <b>abstract</b> number sentences:  <math>10 \div 2 = 5</math>            Grouping            How many 2s are in 10? What is 10 grouped into twos?            ... using <b>concrete</b> equipment:              Count how many groups = 5</p> <p>... using <b>pictorial</b> representations:  </p>

...using **pictorial** representations:

(Number line)



(Counters)

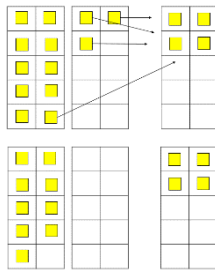


... using **abstract** mental strategies:

(Counting on)

"put 12 in your head and count on 5"

13, 14, 15, 16, 17

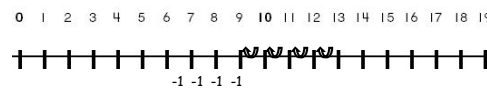


(Bead Strings)

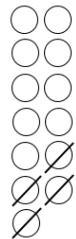


...using **pictorial** representations:

(Number Line)



(Counters)



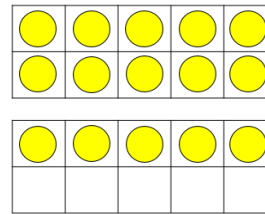
...using **abstract** mental strategies:

(Counting back)

"put 13 in your head and count back 4"

12, 11, 10, 9

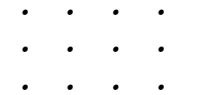
(Arrays and ten frames)



...using

**pictorial** representations:

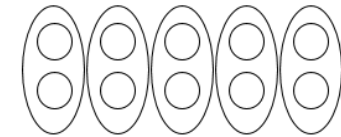
(Arrays)



... using **abstract** mental strategies:


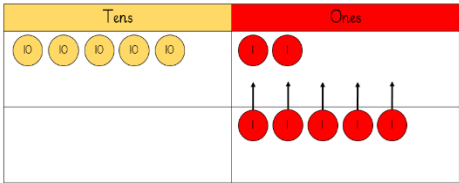
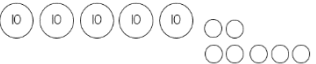

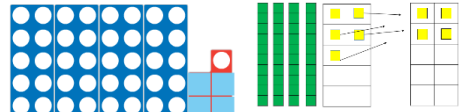
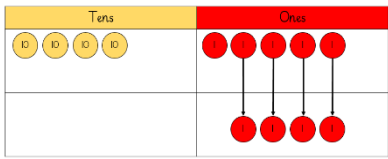

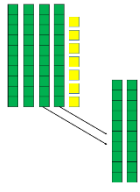
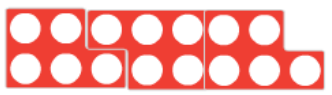

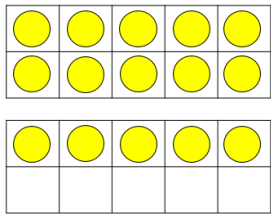

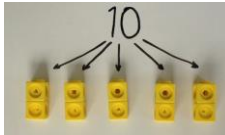

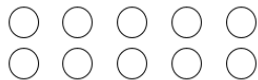
(Counting in multiples)

5 10 15 or 2, 4, 6

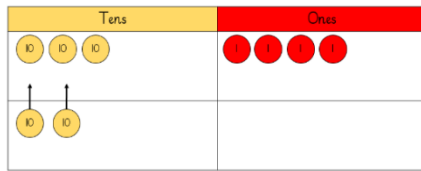


... using **abstract** number sentences:

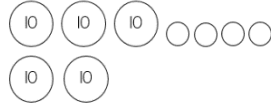
$$10 \div 2 = 5$$

<p>2</p>	<ul style="list-style-type: none"> <li>• Add numbers, including:             <ul style="list-style-type: none"> <li>- a <b>two-digit number and ones</b></li> <li>- a <b>two-digit number and tens</b></li> <li>- two <b>two-digit numbers</b></li> <li>- adding <b>three one-digit numbers</b></li> </ul> </li> <li>• Show that addition of two numbers can be done in any order (<b>commutative</b>).</li> </ul>	<ul style="list-style-type: none"> <li>• Subtract numbers, including:             <ul style="list-style-type: none"> <li>- a <b>two-digit number and ones</b></li> <li>- a <b>two-digit number and tens</b></li> <li>- two <b>two-digit numbers</b></li> </ul> </li> <li>• Show that subtraction of two numbers cannot be done in any order.</li> </ul>	<ul style="list-style-type: none"> <li>• <b>Calculate</b> multiplication statements within the <b>2, 5 and 10 multiplication tables</b> and write them using the multiplication (<math>\times</math>) and equals (<math>=</math>) signs.</li> <li>• Show that multiplication of two numbers can be done in any order (<b>commutative</b>).</li> </ul>	<ul style="list-style-type: none"> <li>• <b>Calculate</b> division statements within the <b>2, 5 and 10 multiplication tables</b> and write them using the division (<math>\div</math>) and equals (<math>=</math>) signs.</li> <li>• Show that division of numbers cannot be done in any order.</li> </ul>
	<p>Addition of a two-digit number and ones:  <math>52 + 5 = 57</math>          ...using <b>concrete</b> equipment:          (Numicon) (Dienes)</p>  <p>(Place value counters)</p>  <p>...using <b>pictorial</b> representations:</p>  <p>Addition of a two-digit number and tens  <math>34 + 20 = 54</math>          ...using <b>concrete</b> equipment:          (Numicon) (Dienes)</p> 	<p>Subtraction of a two-digit number and ones  <math>45 - 4 = 41</math>          ... using <b>concrete</b> equipment:          (Numicon) (Dienes)</p>  <p>(Place value counters)</p>  <p>...using <b>pictorial</b> representations:</p>  <p>Subtraction of a two-digit number and tens  <math>47 - 20 = 27</math>          ...using <b>concrete</b> equipment:          (Numicon) (Dienes)</p> 	<p>Multiplication of two numbers within the 2, 3, 5, 10 multiplication tables.          Introduce <math>\times</math> sign to mean 'how many time' and model recording calculations  <math>5 \times 3 = 15</math> or <math>5, 3</math> times <math>= 15</math>          Understand multiplication can be done in any order <math>3 \times 5 = 15</math> and <math>5 \times 3 = 15</math>.          ... using <b>concrete</b> equipment          (Numicon)</p>  <p>I then use my 10s checker</p>  <p>(Arrays and ten frames)</p>  <p>(Counters - one to many correspondence)</p> <p>1) Because I am counting in multiples of 5, I need to write 5 on my counters. I need three counters.</p> 	<p>Division of numbers within known multiplication tables  <i>Consolidate understanding of 'sharing' and 'grouping' as outlined within Year 1.</i>          Grouping          How many 2s are in 10? What is 10 grouped into twos?          ...using <b>concrete</b> equipment:</p>  <p>Count how many groups = 5</p> <p>(Counters - one to many correspondence)</p> <p>1) Because I am counting in multiples of 2, I need to write 2 on my counters. I need as many counters as it takes me to count in multiples of 2 to get to 10 e.g. 2, 4, 6, 8, 10.</p>  <p>2) Now, I need to point at each counter and count how many groups I have e.g. 1, 2, 3, 4, 5.</p> <p>... using <b>pictorial</b> representations:</p> <p>1 2 3 4 5</p> 

(Place value counters)



...using **pictorial** representations:

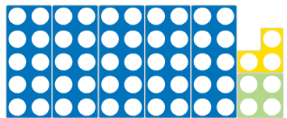
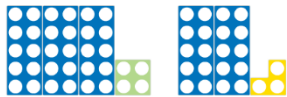


Addition of two two-digit numbers (no exchange):

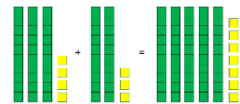
$$34 + 23 = 57$$

...using **concrete** equipment:

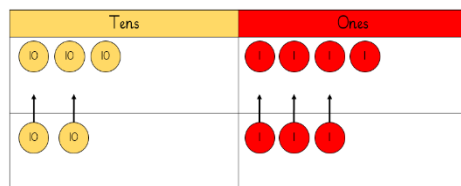
(Numicon)



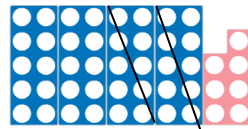
(Dienes)



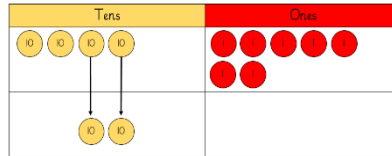
(Place value counters)



...using **pictorial** representations:



(Place value counters)



... using **pictorial** representations:



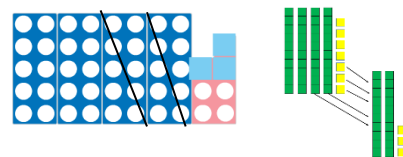
Subtraction two two-digit numbers (no regrouping)

$$47 - 23 = 24$$

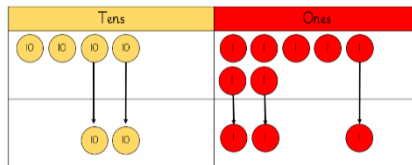
...using **concrete** equipment:

(Numicon)

(Dienes)



(Place value counters)



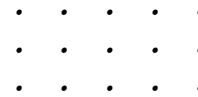
... using **pictorial** representations:



Subtraction two two-digit numbers (regrouping)

2) Now, point at each counter, counting in multiples of 5 e.g. 5, 10, 15.

...using **pictorial** representations: (Arrays)



(Counters - one to many correspondence)

1) I need to write 5 out three times and count '1, 2, 3' as I do this.

5 5 5

2) Now, I need to draw circles around my numbers and count in multiple of 5. E.g. '5, 10, 15'



... using **abstract** mental strategies:

(Counting in multiples)

5 10 15 or 2, 4, 6 or 10, 20, 30

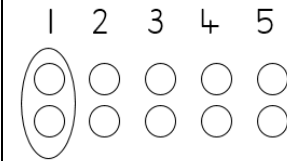


Calculate mathematical statements within the **2, 5 and 10 multiplication tables** and write them using the multiplication ( $\times$ ) and equals ( $=$ ) signs.

$$4 \times 5 = 20$$

$$7 \times 10 = 70$$

$$9 \times 2 = 18$$



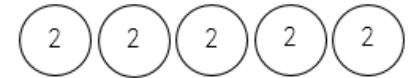
(Counters - one to many)

correspondence)

1) I need to write 2 as many times as it takes me to count in multiples of 2 to get to 10 e.g. 2, 4, 6, 8, 10.

2 2 2 2 2

2) Now, I need to draw circles around my numbers to count how many groups I have e.g. 1, 2, 3, 4, 5.

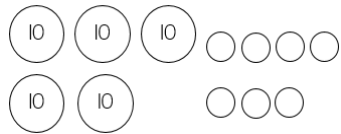


... using **abstract** number sentences:

$$10 \div 2 = 5$$

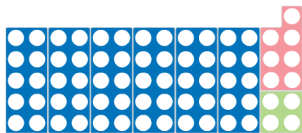
$$12 \div 3 = 4$$

Pupils write number sentences to represent their workings out using the division ( $\div$ ) and equals ( $=$ ) signs.

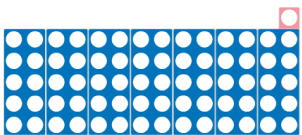


Addition of two two-digit numbers  
(exchange)  
 $47 + 24 = 71$

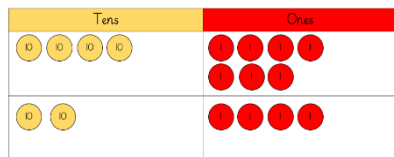
...using **concrete** equipment:  
(Numicon)



Then, I use my 10 checker.



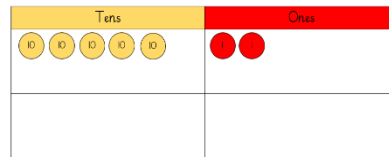
(Place value counters)



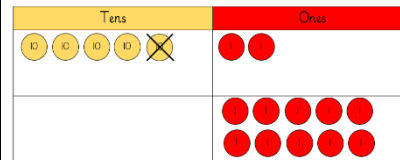
Because I have more than 9 ones, I need to exchange 10 ones for 1 ten.

$$52 - 27 = 25$$

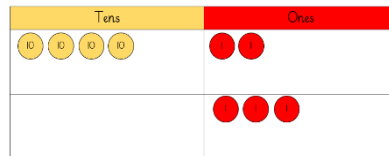
...using **concrete** equipment:  
(Place value counters)



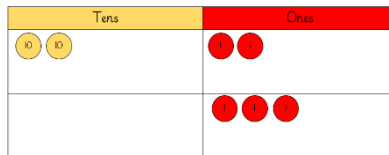
7 ones cannot be subtracted from 2 ones, so exchange 1 ten with 10 ones.



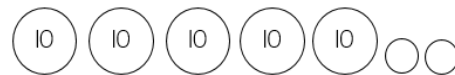
Now, subtract 7 ones.



Now, subtract 2 tens.



...using **pictorial** representations:



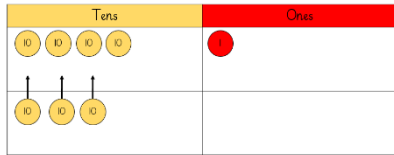
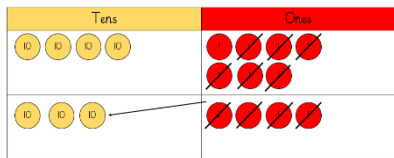
7 ones cannot be subtracted from 2 ones so exchange 1 ten with 10 ones.



Now, subtract 7 ones.



Now, subtract 2 tens.



Following the **concrete** equipment and **pictorial** representations, children will use **abstract**, mental strategies:

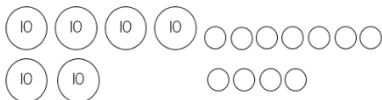
$$45 - 4 = 41$$

$$47 - 20 = 27$$

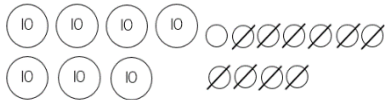
$$47 - 23 = 24$$

$$52 - 27 = 25$$

...using **pictorial** representations:



Because I have more than 9 ones, I need to exchange 10 ones for 1 ten.



Following the **concrete** equipment and **pictorial** representations, children will use **abstract** mental strategies:

$$52 + 5 = 57$$

$$34 + 20 = 54$$

$$34 + 23 = 57$$

$$47 + 24 = 71$$

Addition of three single digit numbers:

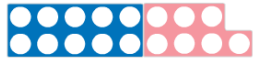
$$4 + 7 + 6 = 17$$

... using **concrete** equipment:



Identify number bonds if possible, e.g. 4 and 6 make 10 /  $4 + 6 = 10$ . Then, add on 7

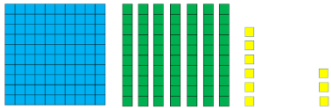
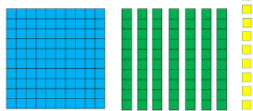
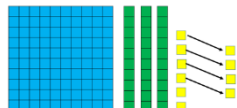
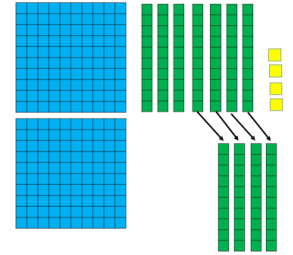
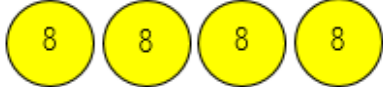

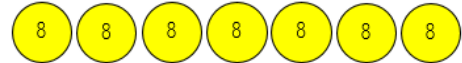

(Numicon)

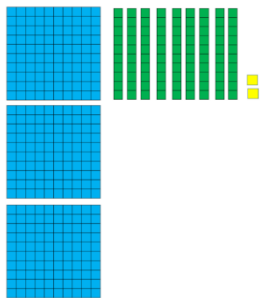
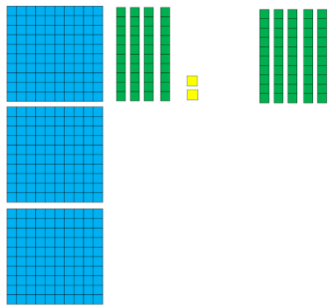


... using **abstract**, mental strategies:

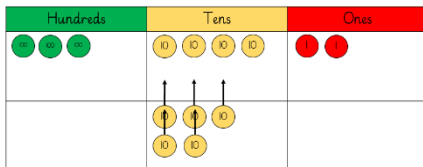
$$\textcircled{4} + 7 + 6 \textcircled{=} 17$$

Identify the two numbers that make ten and then add on the remaining number mentally.

<p>3</p>	<ul style="list-style-type: none"> <li>Add numbers mentally, including:             <ul style="list-style-type: none"> <li>a <b>three-digit number and ones</b></li> <li>a <b>three-digit number and tens</b></li> <li>a <b>three-digit number and hundreds</b></li> </ul> </li> <li>Add numbers with <b>up to three digits</b>, using <b>formal written methods of columnar addition</b></li> </ul>	<ul style="list-style-type: none"> <li>Subtract numbers mentally, including:             <ul style="list-style-type: none"> <li>a <b>three-digit number and ones</b></li> <li>a <b>three-digit number and tens</b></li> <li>a <b>three-digit number and hundreds</b></li> </ul> </li> <li>Subtract a <b>two-digit or 3-digit number from a two-digit or 3 digit number</b> using a <b>formal written method</b></li> </ul>	<ul style="list-style-type: none"> <li>Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.</li> <li>Multiply using multiplication tables that they know, including for <b>two-digit numbers times one-digit numbers</b>, using <b>efficient written methods- 'partitioning method'</b></li> </ul>	<ul style="list-style-type: none"> <li>Recall and use division facts for the 3, 4 and 8 multiplication tables.</li> <li>Divide using known multiplication tables, including for <b>two-digit numbers divided by one-digit numbers</b>, using mental methods, progressing to <b>efficient written methods</b></li> </ul>												
	<p>Addition of a three-digit number and ones:  <math>176 + 3 = 179</math>        ... using <b>concrete</b> equipment:        (Dienes)</p>   <p>(Place value counters)</p> <table border="1" data-bbox="197 949 600 1109"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>●</td> <td>●● ●● ●● ●●</td> <td>●● ●● ●● ●● ●● ●●</td> </tr> </tbody> </table> <p>Addition of a three-digit number and tens:  <math>342 + 50 = 392</math>        ... using <b>concrete</b> equipment:        (Dienes)</p>	Hundreds	Tens	Ones	●	●● ●● ●● ●●	●● ●● ●● ●● ●● ●●	<p>Subtraction of a three-digit number and ones:  <math>136 - 4 = 132</math>        ...using <b>concrete</b> equipment:        (Dienes)</p>  <p>(Place value counters)</p> <table border="1" data-bbox="683 798 1108 965"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>●</td> <td>●● ●● ●●</td> <td>●● ●● ●● ●● ●●</td> </tr> </tbody> </table> <p>Subtraction of a three-digit number and tens:  <math>273 - 40 = 233</math>        ...using <b>concrete</b> equipment:        (Dienes)</p> 	Hundreds	Tens	Ones	●	●● ●● ●●	●● ●● ●● ●● ●●	<p>Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.  <math>8 \times 4 = 32</math>        ... using <b>concrete</b> equipment        (Counters - one to many correspondence)</p> <p>1) Because I am counting in multiples of 8, I need to write 8 on my counters. I need four counters.</p>  <p>2) Now, point at each counter, counting in multiples of 8 e.g. 8, 16, 24, 32.</p> <p>...using <b>pictorial</b> representations:        (Counters - one to many correspondence)</p> <p>1) I need to write 8 out four times and count '1, 2, 3, 4' as I do this.</p> <p style="text-align: center;"><b>8 8 8 8</b></p> <p>2) Now, I need to draw circles around my numbers and count in multiple of 8. E.g. '8, 16, 24, 32'</p>  <p>... using <b>abstract</b> mental strategies:        (Counting in multiples)</p> <p>3, 6, 9... or 4, 8, 12... or 8, 12, 16...</p>	<p>Recall and use division facts for the 3, 4 and 8 multiplication tables.  <math>56 \div 8 = 7</math>        ... using <b>concrete</b> equipment        (Counters - one to many correspondence)</p> <p>1) Because I am counting in multiples of 8, I need to write 8 on my counters. I need as many counters as it takes me to count in multiples of 8 to get to 56 e.g. 8, 16, 24, 32,</p>  <p>40. 48, 56.</p> <p>2) Now, I need to point at each counter and count how many groups I have e.g. 1, 2, 3, 4, 5, 6, 7.</p> <p>...using <b>pictorial</b> representations:        (Counters - one to many correspondence)</p> <p>1) I need to write 8 as many times as it takes me to count in multiples of 8 to get to 56 e.g. 8, 16, 24, 32, 40. 48, 56.</p> <p style="text-align: center;"><b>8 8 8 8 8 8 8</b></p> <p>2) Now, I need to draw circles around my numbers to count how many groups I have e.g. 1, 2, 3, 4, 5, 6, 7.</p> 
Hundreds	Tens	Ones														
●	●● ●● ●● ●●	●● ●● ●● ●● ●● ●●														
Hundreds	Tens	Ones														
●	●● ●● ●●	●● ●● ●● ●● ●●														



(Place value counters)



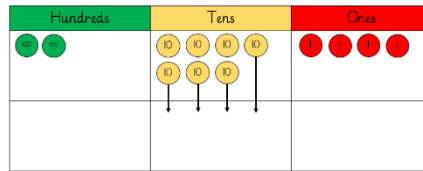
Addition of a three-digit number and hundreds:

$$306 + 300 = 606$$

... using **concrete** equipment:

(Dienes)

(Place value counters)

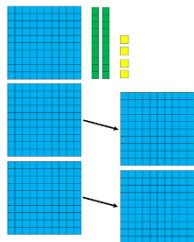


Subtraction of a three-digit number and hundreds:

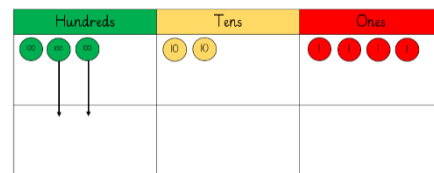
$$324 - 200 = 124$$

... using **concrete** equipment:

(Dienes)



(Place value counters)



Subtraction of numbers with up to three digits

$$263 - 129 = 134$$

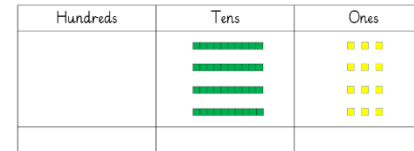
Multiplication of a two-digit number by a one-digit number.

$$13 \times 4 = 52$$

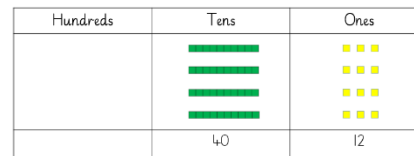
$$24 \times 3 = 72$$

...using **concrete** equipment

(Dienes)



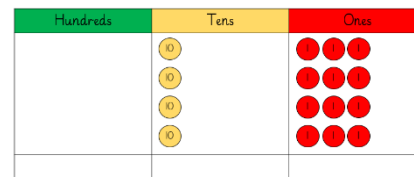
Count the number of ones, and then count the number of tens.



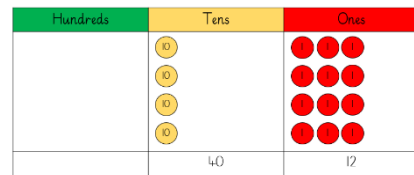
$$40 + 12 = 52$$

(Place value counters)

First calculation



Count the number of ones, and then count the number of tens.



$$40 + 12 = 52$$

Division of a two-digit number by a one-digit number, using known multiplication tables.

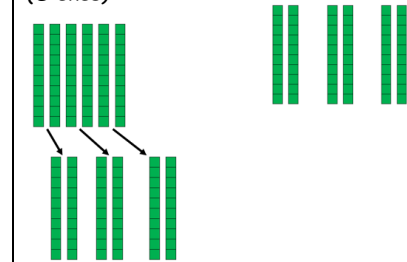
$$60 \div 3 = 20$$

...using **concrete** equipment

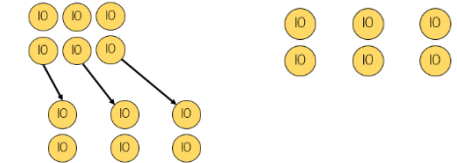
Sharing

Grouping

(Dienes)



(Place value counters)



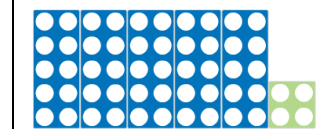
$$6 \text{ tens} \div 3 = 2 \text{ tens} = 20$$

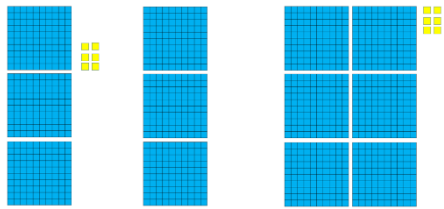
Dividing a two-digit numbers by one-digit numbers.

$$54 \div 3 = 18.$$

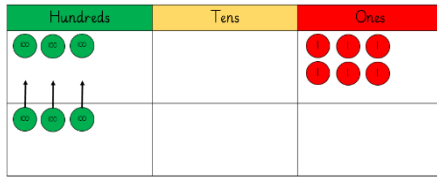
...using **concrete** equipment:

(Numicon)





(Place value counters)

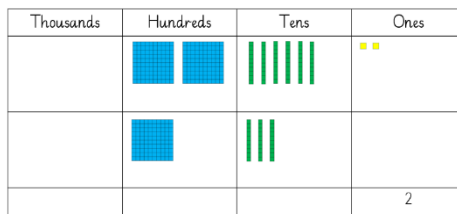
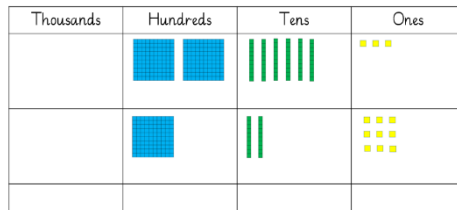


Addition of numbers with up to three digits

$$263 + 119 = 392$$

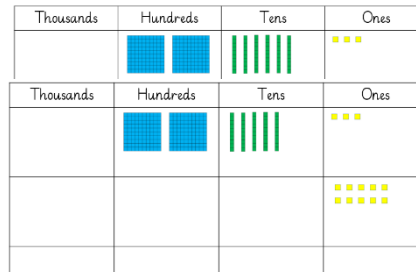
...using concrete equipment:

(Dienes)



...using concrete equipment:

(Dienes)

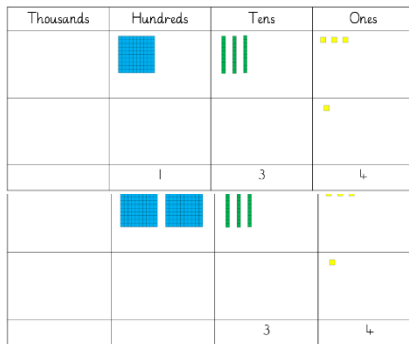


9 ones cannot be subtracted from 3 ones so exchange 1 ten for 10 ones.

Now, subtract 9 ones.

Now, subtract 2 tens.

Now, subtract 1 hundred.

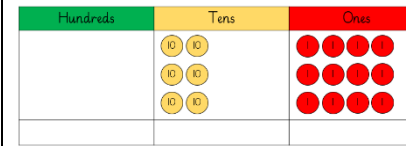


(Place value counters)

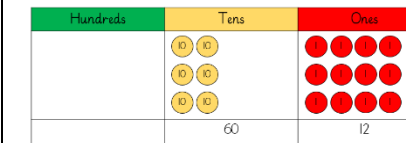


9 ones cannot be subtracted from 3 ones so exchange 1 ten for 10 ones.

Second calculation



Count the number of ones, and then count the number of tens.

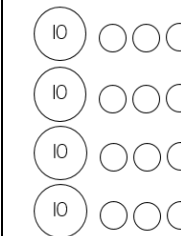


$$60 + 12 = 72$$

...using pictorial representations

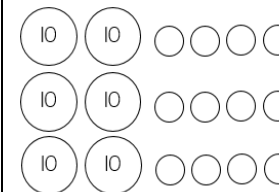
First calculation

Count the ones first, then the tens and add the numbers together.



$$40 + 12 = 52$$

Second calculation



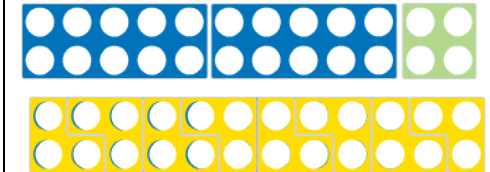
$$60 + 12 = 72$$

...using abstract methods

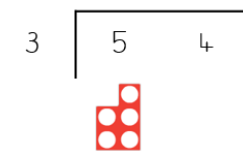
Share the tens equally into 3 groups.



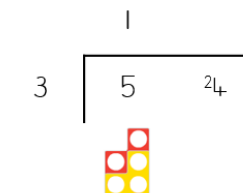
I have 24 left over. Now I need to divide 24 by 3.



(Numicon)



How many 3s goes into 5?



Now, make 24 and check how many 3s go into 24.

Exchange 10 ones for 1 ten.

(Place value counters)

Thousands	Hundreds	Tens	Ones
	100 100	10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1
	100	10 10	1 1 1 1 1 1 1 1 1 1
	100 100	10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1
	100	10 10 10	1 1 1 1 1 1 1 1 1 1
			2
	100 100	10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1
	100	10 10 10	1 1 1 1 1 1 1 1 1 1
	3	9	2

Exchange 10 ones for 1 ten.

Hundreds	Tens	Ones
100 100	10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1
		1 1 1 1 1 1 1 1 1 1

Now, subtract 9 ones.

Hundreds	Tens	Ones
100 100	10 10 10 10 10	1 1 1 1 1 1 1 1
		1

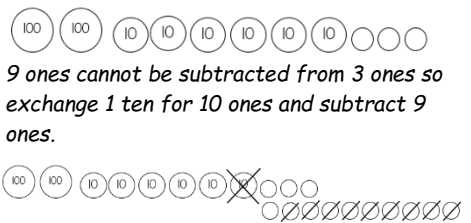
Now, subtract 2 tens.

Hundreds	Tens	Ones
100 100	10 10 10 10 10 10	1 1 1 1 1 1 1 1
	3	4

Now, subtract 1 hundred.

Hundreds	Tens	Ones
100	10 10 10 10 10 10	1 1 1 1 1 1 1 1
1	3	4

...using pictorial representations



Now, subtract 2 tens.

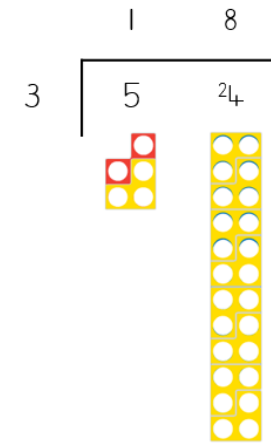
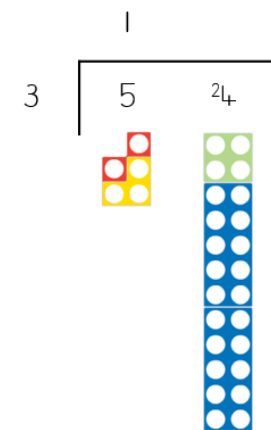


Now, subtract 1 hundred.

Use of partitioning method, independent of equipment and diagrams.

$$13 \times 4 = (10 \times 4) + (3 \times 4) \\ = 40 + 12 \\ = 52$$

$$24 \times 3 = (20 \times 3) + (4 \times 3) \\ = 60 + 12 \\ = 72$$



...using **abstract** methods  
Completion of number sentences.  
 $60 \div 3 = 20$

**Progression in the formal written method for division:**

**Step 1**  
Two-digit number divided by a one-digit number - no exchanging across place value columns e.g.  $84 \div 4 = 21$

...using pictorial representations



Exchange ten ones for 1 ten.

Thousands	Hundreds	Tens	Ones
	● ●	● ● ● ● ● ●	● ●
	●	● ● ●	
		9	2

Thousands	Hundreds	Tens	Ones
	● ●	● ● ● ● ● ●	● ●
	●	● ● ●	
	3	9	2



... using abstract mental strategies

(Column method)

$$\begin{array}{r}
 2 \ 6 \ 3 \\
 + \ 1 \ 2 \ 9 \\
 \hline
 3 \ 9 \ 2 \\
 \hline
 1
 \end{array}$$

Progression in columnar addition:

Step 1 (to introduce)

2 digits - no exchanging e.g. 45 + 32

Step 2

2 digits - exchanging to the tens e.g. 43 + 18



...using abstract mental strategies

(Column method)

$$\begin{array}{r}
 5 \ 1 \\
 2 \ 6 \ 3 \\
 - \ 1 \ 2 \ 9 \\
 \hline
 1 \ 3 \ 4
 \end{array}$$

Progression in columnar subtraction:

Step 1 (to introduce)

2 digits - no exchanging e.g. 58 - 27

Step 2

2 digits - exchanging from tens e.g. 42 - 18

Step 3

3 digits - exchanging from tens e.g. 263 - 119

Step 4

3 digits - exchanging from hundreds e.g. 347 - 261

Step 5

2 from 3 digit numbers - understand place value including the place value of columns.

	2	1
4	8	4

Step 2

Two-digit number divided by a one-digit number - involving exchanging across place value columns without remainders e.g.

	1	8
3	5	24

	<p><b>Step 3</b> 3 digits - exchanging to the tens e.g. 263 + 119</p> <p><b>Step 4</b> 3 digits - exchanging to the hundreds e.g. 357 + 261</p> <p><b>Step 5</b> 3 digits - exchanging to the thousands e.g. 847 + 931</p> <p><b>Step 6</b> 2 and 3 digit numbers - understand place value including the place value of columns.</p>			
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4	<ul style="list-style-type: none"> <li>Add numbers with <b>up to 4 digits</b> using mental strategies and the <b>formal written methods (columnar addition)</b></li> <li>Add numbers with <b>2 decimal places</b>, using <b>formal written methods (columnar addition)</b></li> </ul>	<ul style="list-style-type: none"> <li>Subtract numbers with <b>up to 4 digits</b> using mental strategies and the <b>formal written methods (columnar subtraction)</b></li> <li>Subtract numbers with <b>2 decimal places</b>, using <b>formal written methods (columnar subtraction)</b></li> </ul>	<ul style="list-style-type: none"> <li>Recall multiplication facts for multiplication tables up to <math>12 \times 12</math>.</li> <li>Multiply <b>two-digit and three-digit numbers by a one-digit number</b> using formal written layout e.g. <math>84 \times 6</math>, <math>216 \times 4</math></li> <li>Multiply <b>three-digit numbers with 1 decimal place by a one-digit number</b> using formal written layout e.g. <math>134.5 \times 7</math></li> </ul>	<ul style="list-style-type: none"> <li>Recall division facts for multiplication tables up to <math>12 \times 12</math>.</li> <li>Divide numbers up to <b>3 digits by a 1 digit number</b> using the formal written method (no remainders)</li> </ul>
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<p>Addition of numbers with up to four digits: ...using <b>concrete</b> equipment <i>Use of place value chart and dienes (as used in Year 3).</i></p> <table border="1" data-bbox="197 683 591 863"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td></tr> </tbody> </table> <p>Use of place value chart and place value counters (as used in Year 3).</p> <table border="1" data-bbox="197 938 591 1118"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td></tr> </tbody> </table> <p>...using <b>pictorial</b> representations Use of place value counters to support understanding (as used in Year 3). ...using <b>abstract</b> strategies (Column method) four digit + four digit</p>	Thousands	Hundreds	Tens	Ones													Thousands	Hundreds	Tens	Ones													<p>Subtraction of numbers with up to four digits: ...using <b>concrete</b> equipment <i>Use of place value chart and dienes (as used in Year 3).</i></p> <table border="1" data-bbox="689 683 1084 863"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td></tr> </tbody> </table> <p>Use of place value chart and place value counters (as used in Year 3).</p> <table border="1" data-bbox="689 938 1084 1118"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td><td> </td></tr> </tbody> </table> <p>...using <b>pictorial</b> representations <i>Use of place value counters to support understanding (as used in Year 3).</i> ...using <b>abstract</b> strategies four digit - four digit</p> $\begin{array}{r} 5 \quad 1 \quad 1 \\ \phantom{5} \quad 3 \\ \hline 6 \quad 4 \quad 6 \quad 7 \end{array}$	Thousands	Hundreds	Tens	Ones													Thousands	Hundreds	Tens	Ones													<p>Recall and use multiplication facts for the multiplication tables up to <math>12 \times 12</math>. ...using <b>concrete</b> equipment <i>Use of counters - one to many correspondence (as used in Year 3).</i> ...using <b>pictorial</b> representations <i>Use of counters - one to many correspondence (as used in Year 3).</i> ... using <b>abstract</b> mental strategies: <i>Counting in multiples (the same as year 3 but involving all multiplication facts up to <math>12 \times 12</math>)</i> Multiplication of two and three digit numbers by a one-digit number <math>216 \times 4 = 864</math> ...using <b>concrete</b> equipment (Place value counters)</p> <table border="1" data-bbox="1193 1193 1601 1358"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td> </td> <td>●●●●</td> <td>●●</td> <td>●●●●●●●●</td> </tr> <tr> <td> </td> <td>●●●●</td> <td>●●</td> <td>●●●●●●●●</td> </tr> <tr> <td> </td> <td>●●●●</td> <td>●●</td> <td>●●●●●●●●</td> </tr> </tbody> </table> <p>First, count how many ones there are. Pupils to count in multiples e.g. 6, 12, 18, 24. Because I have '24' ones in one place value column, I</p>	Thousands	Hundreds	Tens	Ones		●●●●	●●	●●●●●●●●		●●●●	●●	●●●●●●●●		●●●●	●●	●●●●●●●●	<p>Recall and use division facts for the multiplication tables up to <math>12 \times 12</math>. ...using <b>concrete</b> equipment <i>Use of counters - one to many correspondence (as used in Year 3).</i> ...using <b>pictorial</b> representations <i>Use of counters - one to many correspondence (as used in Year 3).</i> ... using <b>abstract</b> mental strategies: <i>Counting in multiples (the same as year 3 but involving all division facts up to <math>12 \times 12</math>)</i> Divide numbers with up to three-digit by a one-digit number <math>976 \div 8 = 122</math> ...using <b>concrete</b> equipment (Numicon)</p> <table border="1" data-bbox="1742 1150 1984 1286"> <tr> <td>8</td> <td>9</td> <td>7</td> <td>6</td> </tr> <tr> <td>●●●●●●●●</td> <td> </td> <td> </td> <td> </td> </tr> </table>	8	9	7	6	●●●●●●●●			
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$$\begin{array}{r}
 1\ 4\ 5\ 6 \\
 +\ 7\ 6\ 5 \\
 \hline
 2\ 2\ 2\ 1 \\
 \hline
 1\ 1\ 1 \\
 \\
 4\ 4\ 7\ 8 \\
 +\ 3\ 7\ 6\ 2 \\
 \hline
 8\ 2\ 4\ 0 \\
 \hline
 1\ 1\ 1
 \end{array}$$

four digit + three digits  
Understanding place value and the place value of columns

Using 0 as a place holder

$$\begin{array}{r}
 2\ 6\ 0\ 5 \\
 +\ 8\ 0\ 9 \\
 \hline
 3\ 4\ 1\ 4 \\
 \hline
 1\ 1
 \end{array}$$

$$\begin{array}{r}
 -\ 2\ 6\ 8\ 4 \\
 \hline
 3\ 7\ 8\ 3
 \end{array}$$

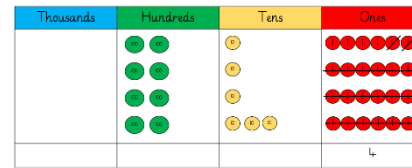
four digit - three digit  
Understanding place value and the place value of columns

$$\begin{array}{r}
 1\ 19\ 19\ 1 \\
 2\ 0\ 0\ 0 \\
 -\ 4\ 7\ 5 \\
 \hline
 1\ 5\ 2\ 5 \\
 \\
 1\ 1\ 1\ 1 \\
 4\ 3 \\
 \cancel{2}\ \cancel{5}\ \cancel{4}\ 3 \\
 -\ 8\ 7\ 6 \\
 \hline
 1\ 6\ 6\ 7
 \end{array}$$

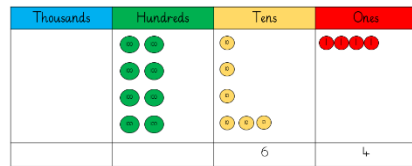
Using 0 as a place holder

$$\begin{array}{r}
 5\ 19\ 1 \\
 2\ 0\ 5 \\
 -\ 8\ 9 \\
 \hline
 2\ 5\ 1\ 6
 \end{array}$$

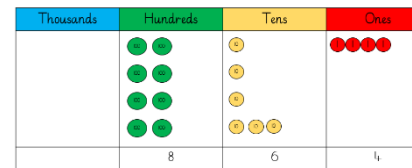
know I need to exchange 20 ones for 2 tens and count how many ones are left.



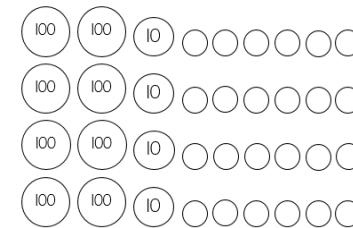
Now, count how many tens there are.



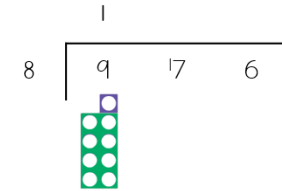
Now, count how many hundreds there are. Pupils to count in multiples. E.g. '2, 4, 6, 8'



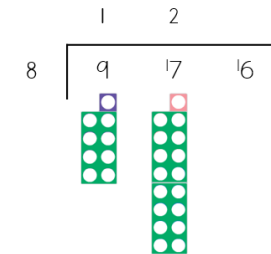
...using pictorial representations



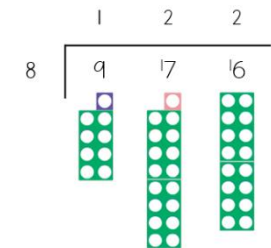
How many 8s go into 9?



Now, make 17 and check how many 8s go into 17.



Now, make 16 and check how many 8s go into 16.



...using abstract methods

Progression in the formal written method for division:

Numbers with 1 decimal place

$$\begin{array}{r} 37.93 \\ + 20.35 \\ \hline 58.28 \\ 1 \end{array}$$

Numbers with 2 decimal places

\*Use partitioning methods to support understanding of columnar addition where appropriate.

$$\begin{array}{r} 37.934 \\ + 20.352 \\ \hline 58.286 \\ 1 \end{array}$$

Numbers with 1 decimal place

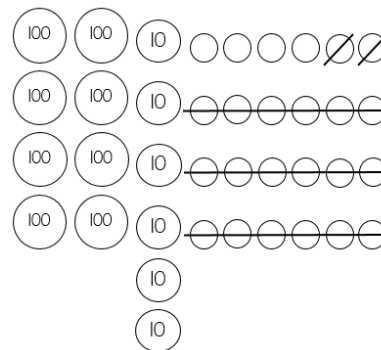
$$\begin{array}{r} 31 \\ 74.37 \\ - 21.62 \\ \hline 52.75 \end{array}$$

Numbers with 2 decimal places

$$\begin{array}{r} 31 \\ 74.372 \\ - 21.621 \\ \hline 52.751 \end{array}$$

\*Use partitioning methods to support understanding of columnar subtraction where appropriate.

First, count how many ones there are. Pupils to count in multiples e.g. 6, 12, 18, 24. Because I know I cannot have '24' ones in one place value column, I know I need to exchange 20 ones for 2 tens and count how many ones are left.



Now, count how many tens there are and how many hundreds there are. Pupils to count in multiples e.g. 2, 4, 6, 8.

...using abstract methods

**Progression in column multiplication:**

**Step 1** (to introduce)

two digits x one digit - no exchanging e.g.  $32 \times 3$

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 96 \end{array}$$

**Step 2**

two digits x one digit - exchange to tens e.g.  $23 \times 4$

(Expand to model exchanging)

(Sometimes new arrivals arrive knowing the expanded version)

$$\begin{array}{r} 23 \\ \times 2 \\ \hline 46 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

**Step 1**

Two and three-digit numbers divided by a one-digit number - no exchanging across place value columns e.g.  $84 \div 4 = 21$ ,  $396 \div 3 = 132$

$$\begin{array}{r} 21 \\ 4 \overline{)84} \\ \underline{84} \\ 0 \end{array}$$

$$\begin{array}{r} 132 \\ 3 \overline{)396} \\ \underline{396} \\ 0 \end{array}$$

**Step 2**

Two and three-digit numbers divided by a one-digit number - involving exchanging across place value columns without remainders e.g.  $138 \div 6 = 23$ ,  $976 \div 8 = 122$

$$\begin{array}{r} 23 \\ 6 \overline{)138} \\ \underline{12} \phantom{0} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

$$\begin{array}{r} 122 \\ 8 \overline{)976} \\ \underline{8} \phantom{0} \phantom{0} \\ 17 \phantom{0} \\ \underline{16} \phantom{0} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

\* Introduce the concept of a remainder.

$$\begin{array}{r}
 \times \quad 4 \\
 \hline
 9 \quad 2 \\
 \hline
 1
 \end{array}
 \qquad
 \begin{array}{r}
 \times \quad 4 \\
 \hline
 1 \quad 2 \\
 + \quad 8 \quad 0 \\
 \hline
 9 \quad 2
 \end{array}$$

**Step 3**

two digits x one digit - exchange to tens and hundreds e.g. 84 x 6

$$\begin{array}{r}
 8 \quad 4 \\
 \times \quad 6 \\
 \hline
 5 \quad 0 \quad 4 \\
 \hline
 5 \quad 2
 \end{array}
 \qquad
 \begin{array}{r}
 8 \quad 4 \\
 \times \quad 6 \\
 \hline
 2 \quad 4 \\
 + \quad 4 \quad 8 \quad 0 \\
 \hline
 5 \quad 0 \quad 4
 \end{array}$$

**Step 4**

three digits x one digit - exchange to tens e.g. 219 x 4

$$\begin{array}{r}
 2 \quad 1 \quad 9 \\
 \times \quad 4 \\
 \hline
 8 \quad 7 \quad 6 \\
 \hline
 3
 \end{array}$$

**Step 5**

three digits x one digit - exchange to tens, hundreds and thousands e.g. 425 x 4

$$\begin{array}{r}
 4 \quad 2 \quad 5 \\
 \times \quad 4 \\
 \hline
 1 \quad 8 \quad 0 \quad 0 \\
 \hline
 1 \quad 2 \quad 2
 \end{array}$$

5	<ul style="list-style-type: none"> <li>• Add <b>whole numbers with more than 4 digits</b> (and with <b>up to 3 decimal places</b>), including using formal written methods (columnar addition)</li> </ul>	<ul style="list-style-type: none"> <li>• Subtract <b>whole numbers with more than 4 digits</b> (and with <b>up to 3 decimal places</b>), including using formal written methods (columnar subtraction)</li> </ul>	<ul style="list-style-type: none"> <li>• Multiply numbers <b>up to 4 digits by a 1 digit number</b> using a formal written method <i>e.g.</i> <math>3721 \times 7</math></li> <li>• Multiply <b>one-digit numbers with up to three decimal places by whole numbers</b></li> <li>• Multiply numbers <b>up to 4 digits by 2-digit number</b> using a formal written method <i>e.g.</i> <math>3721 \times 37</math></li> </ul>	<ul style="list-style-type: none"> <li>• Divide numbers <b>up to 4 digits by a one-digit number</b> using the formal written method and <b>interpret remainders</b></li> <li>• Divide numbers <b>up to 4 digits with up to 3 decimal places by a one-digit number</b> using the formal short written method</li> </ul>																
	<p>The same as Year 4 but with larger numbers and with a greater number of decimal places - up to 3 decimal places. Continue to ensure that the use of '0' as a placeholder is used to ensure pupils are confident with the exchanging and adding on process.</p>	<p>The same as Year 4 but with larger numbers and with a greater number of decimal places - up to 3 decimal places. Continue to ensure that the use of '0' as a placeholder is used to ensure pupils are confident with the exchanging process.</p>	<p>Multiplication of a four-digit numbers by a one-digit numbers.</p> <p>...using <b>concrete</b> equipment <i>Use of place value counters (as used in Year 4).</i></p> <p>...using <b>pictorial</b> representations <i>Use of place value counters (as used in Year 4).</i></p> <p>... using <b>abstract</b> methods:</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right; padding-right: 20px;"> <math display="block">\begin{array}{r} 3721 \\ \times \quad 7 \\ \hline 26047 \\ 251 \end{array}</math> </td> <td style="text-align: right;"> <math display="block">\begin{array}{r} 4725 \\ \times \quad 9 \\ \hline 42525 \\ 4624 \end{array}</math> </td> </tr> </table> <p>Multiplication of a one-digit number with up to three decimal places by a one-digit number.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right; padding-right: 20px;"> <math display="block">\begin{array}{r} 1.43 \\ \times \quad 6 \\ \hline 8.58 \\ 2 \quad 1 \end{array}</math> </td> <td></td> </tr> </table> <p>Develop to up to 4 digits with up to 3 decimal places by a one-digit number.</p> <p>Multiplication of a four-digit number by a two-digit number.</p>	$\begin{array}{r} 3721 \\ \times \quad 7 \\ \hline 26047 \\ 251 \end{array}$	$\begin{array}{r} 4725 \\ \times \quad 9 \\ \hline 42525 \\ 4624 \end{array}$	$\begin{array}{r} 1.43 \\ \times \quad 6 \\ \hline 8.58 \\ 2 \quad 1 \end{array}$		<p>Division of numbers with up to four digits by a one-digit number.</p> <p><i>Consolidate understanding of using the formal written method without remainders as outlined within Year 4.</i></p> <p>...using <b>concrete</b> equipment <i>Use of Numicon (as used in Year 4)</i></p> <p>...using <b>abstract</b> methods</p> <p><b>Progression in the formal written method for division:</b></p> <p><b>Step 1</b></p> <p>Two-digit number divided by one-digit number - with remainders</p> <p><math>76 \div 6 = 12 \text{ r } 4</math></p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">r</td> <td style="border: 1px solid black; padding: 2px;">4</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;"></td> <td style="border: 1px solid black; padding: 2px;"></td> <td style="border: 1px solid black; padding: 2px;"></td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;"></td> <td style="border: 1px solid black; padding: 2px;"></td> </tr> </table> <p><b>Step 2</b></p> <p>Three-digit number divided by one-digit number - with remainders</p> <p><math>852 \div 7 = 121 \text{ r } 5</math>.</p> <p>Round up or down given the context of the problem.</p>	6	7	1	2	r	4				6		
$\begin{array}{r} 3721 \\ \times \quad 7 \\ \hline 26047 \\ 251 \end{array}$	$\begin{array}{r} 4725 \\ \times \quad 9 \\ \hline 42525 \\ 4624 \end{array}$																			
$\begin{array}{r} 1.43 \\ \times \quad 6 \\ \hline 8.58 \\ 2 \quad 1 \end{array}$																				
6	7	1	2	r	4															
			6																	

$$\begin{array}{r}
 3701 \\
 \times 37 \\
 \hline
 25907 \\
 + 111030 \\
 \hline
 136937
 \end{array}$$

	1	2	1	r	5
7	8	<sup>1</sup> 5	<sup>1</sup> 2		

**Step 3**

Up to four-digits with up to 3 decimal places by a one-digit number

		2	4	•	9
7	1	7	<sup>3</sup> 4	•	<sup>6</sup> 3

		2	3	•	2	9
8	1	8	<sup>2</sup> 6	•	<sup>2</sup> 3	<sup>7</sup> 2

**Step 4**

Four-digit number divided by one-digit number - with remainders- interpreted as a decimal (to 3 decimal places)

$$6497 \div 8 = 812.125$$

	0	8	1	2	•	1	2	5
8	6	<sup>6</sup> 4	9	<sup>17</sup>	•	<sup>10</sup>	<sup>20</sup>	<sup>40</sup>

6	<ul style="list-style-type: none"> <li>Add multi-digit numbers with more than 4 digits (with up to 3 decimal places), using formal written methods (columnar addition)</li> </ul>	<ul style="list-style-type: none"> <li>Subtract multi-digit numbers with more than 4 digits (with up to 3 decimal places), using formal written methods (columnar subtraction)</li> </ul>	<ul style="list-style-type: none"> <li>Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</li> </ul>	<ul style="list-style-type: none"> <li>Divide numbers up to 4 digits (with up to 3 decimal places) by a two-digit whole number using the formal written method of division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context - Short division / Long division</li> </ul>														
	<p>The same as Year 4 and 5 but with multi-digit numbers with more than 4 digits (and with up to 3 decimal places).</p>	<p>The same as Year 4 and 5 but with multi-digit numbers with more than 4 digits (and with up to 3 decimal places).</p>	<p>Multiplication of a four-digit number by a two-digit number.</p> $  \begin{array}{r}  3701 \\  \times \quad 37 \\  \hline  25907 \\  + 111030 \\  \hline  136937  \end{array}  $	<p>Consolidate understanding of using the formal written method for dividing three-digit number with up to 3 decimal places by one-digit number as outlined in Year 5.</p> <p>Division of numbers with up to four-digits and three decimal places, by a two-digit whole number.</p> <p><math>4138 \div 17 = 243 \text{ r } 7</math></p> <p>...using <b>concrete</b> equipment</p> <p>Use of Numicon (as used in Year 4 and Year 5)</p> <p>...using <b>abstract</b> methods</p> <p>Short Division</p> <table border="1" data-bbox="1671 839 2063 935"> <tr> <td></td> <td></td> <td></td> <td>2</td> <td>4</td> <td>3</td> <td>r 7</td> </tr> <tr> <td>1</td> <td>7</td> <td>4</td> <td><sup>4</sup>1</td> <td><sup>7</sup>3</td> <td><sup>5</sup>8</td> <td></td> </tr> </table> <p>= 243 remainder 7 or 243 r 7 or 243 <math>\frac{7}{17}</math> or 243.41 or 243 (to the nearest whole number)* Answer according to the question.</p> <p>Long Division</p> $  \begin{array}{r}  243 \text{ r } 7 \\  17 \overline{) 4138} \\  \underline{34} \phantom{00} \\  73 \phantom{00} \\  \underline{68} \phantom{00} \\  58 \phantom{00} \\  \underline{51} \phantom{00} \\  7  \end{array}  $				2	4	3	r 7	1	7	4	<sup>4</sup> 1	<sup>7</sup> 3	<sup>5</sup> 8	
			2	4	3	r 7												
1	7	4	<sup>4</sup> 1	<sup>7</sup> 3	<sup>5</sup> 8													

